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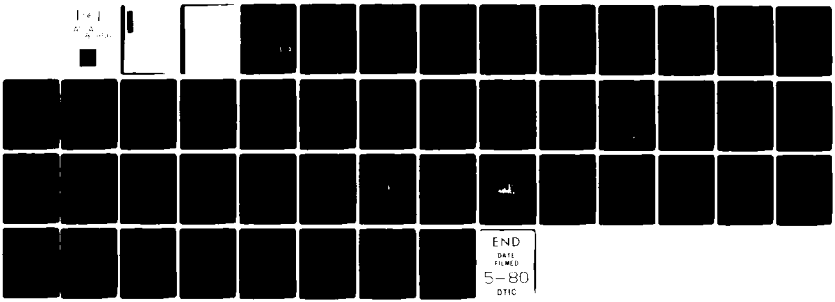

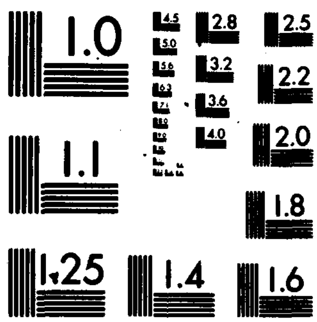

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

A NEW ALGORITHM FOR ADAPTIVE RADAR SIGNAL PROCESSING

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ABSTRACT

A new adaptive processing algorithm, called Weighted Covariance Estimation (WCE), for the detection of signals in interference of unknown character is presented. Its main advantage over present techniques, such as Sample Matrix Inversion (SMI), is its tendency not to suppress desired signals present in the learning data.

WCE and SMI are compared for a radar problem of practical interest, adaptive MTI from a moving platform, using both simulated and actual radar data.

CONTENTS

INTRODUCTION

Adaptive array processing[1] is a powerful technique for detecting known signals immersed in a Gaussian interference background. The statistics of Gaussian interference are completely characterized by a covariance matrix which, together with the signal, determines the optimum receiver structure. If the covariance matrix is not known, it must be estimated from the available observations.

The problem of interest here is the adaptive detection of radar returns from moving targets in the presence of ground clutter and receiver noise. The radar return from each transmitted signal is received by M sensors and matched-filtered. The resulting outputs are sampled at the Nyquist rate, yielding complex samples in N range cells from each sensor. One (vector) observation consists of the M complex samples from a particular range cell. To obtain the necessary number of independent observations of the interference process, it is commonly assumed that the interference covariance matrix is the same in every range cell. The maximum likelihood estimate of the covariance matrix is then the sample covariance matrix[2], which is inverted and used to determine the optimum receiver weighting[3]. This technique has been called Sample Matrix Inversion (SMI).

This paper proposes a different model for the interference statistics and derives the maximum likelihood estimate of the interference covariance matrix. This estimate is more expensive computationally than the sample covariance, but offers greater immunity to the suppression of desired signals.

The performance of the algorithm is compared with that of SMI using both simulated data and real data from an airborne MTI radar.

Adaptive Detection

The problem of detecting a (complex) signal in colored Gaussian interference given K (scalar) complex observations may be stated as follows: Under hypothesis H_o (no signal present), the observations are of the form

$$H_o: \quad z_k = n_k \qquad k = 1, \ldots K$$

while under hypothesis H_1 (signal present) they take the form

$$H_1: \quad z_k = \beta s_k(\underline{a}) + n_k$$

Here β is an unknown complex number, acknowledging the fact that the amplitude and phase of the signal are unknown. Any additional unknown parameters are contained in the vector argument $\underline{a}$. Under the Gaussian assumption, the interference is completely characterized by its first- and second-order statistics[†]

$$E(n_k) = E(n_k n_\ell) = 0$$

$$E(n_k n_\ell^*) = \Lambda_{k\ell}$$

The detection strategy is based on the generalized likelihood ratio test

$$\ell(\underline{z}) = \frac{\max\limits_{\beta,\underline{a}} \; p(\underline{z}|\beta,\underline{a},H_1)}{p(\underline{z}|H_o)} \; \underset{H_o}{\overset{H_1}{\gtrless}} \; \lambda$$

It can be shown that the test reduces to

$$\max_{\underline{a}} \; |\underline{s}^*(\underline{a})\Lambda^{-1}\underline{z}| \; \underset{H_o}{\overset{H_1}{\gtrless}} \; \lambda' \qquad (1)$$

Thus each observation z_k is multiplied by a complex weight w_k^*, the resulting complex numbers are summed, and the magnitude of the resultant is compared with a threshold. The weighting vector $\underline{w}$ is given by

$$\underline{w} = \Lambda^{-1}\underline{s}(\hat{\underline{a}})$$

† * denotes the complex conjugate of a scalar and the complex conjugate transpose of a vector or matrix.

where $\hat{\underline{a}}$ is the vector of parameter values that maximizes (1).

When some or all of the elements of Λ are unknown, the test becomes

$$\ell(\underline{z}) = \frac{\max\limits_{\beta,\underline{a},\Lambda_u} p(\underline{z}|\beta,\underline{a},\Lambda_u,H_1)}{\max\limits_{\Lambda_u} p(\underline{z}|\Lambda_u,H_o)}$$

where Λ_u denotes the set of unknown elements of Λ. Evidently, if the number of (real) unknown parameters exceeds the number of (real) observations, the observation space cannot be mapped 1:1 into the parameter space. Additional independent observations are required in order to form meaningful estimates.

In most practical detection problems, the presence of a signal is an unlikely event. Multiple independent observations under H_1 are thus not available. On the other hand, the interference environment is often statistically stationary (or quasi-stationary) in time and/or space, so that additional independent observations under H_o (no signal) are easily obtained. In such cases, it is reasonable to estimate interference statistics assuming that H_o is true, and use the results to estimate signal parameters.

Specifically, if N independent observations $\underline{z}_1, \ldots \underline{z}_N$ are available under H_o, find the maximum likelihood estimate $\hat{\Lambda}_u$ of all the unknown statistical parameters. Given another independent observation $\underline{z}_{N+1}$, use

$$\frac{\max\limits_{\beta,\underline{a}} p(\underline{z}_{N+1}|\beta,\underline{a},\hat{\Lambda}_u,H_1)}{p(\underline{z}_{N+1}|\hat{\Lambda}_u,H_o)}$$

as the detection statistic to decide whether or not a signal is present in $\underline{z}_{N+1}$. In reality, of course, any of the $\underline{z}_n$ may contain a signal, so the estimate of the interference statistics is based on all N+1 observations, and used to test each observation for the presence of a signal. The hope is that the presence of a few signals in the N+1 observations will not seriously perturb the estimates of interference statistics.

The Sample (covariance) Matrix Inversion (SMI) technique[3] is a special case of this adaptive detection strategy in which the entire interference covariance matrix is unknown, but assumed constant for all observations. Goodman shows[2] that the maximum likelihood estimate of the covariance matrix is the sample covariance matrix

$$\hat{\Lambda} = \frac{1}{N} \sum_{n=1}^{N} \underline{z}_n \underline{z}_n^*$$

and derives the joint characteristic function and probability density of its N^2 real random variables.

This paper proposes an alternative model in which the interference covariance matrix for the n^{th} independent observation is of the form

$$\Lambda_n = \sigma_n^2 C \qquad (2)$$

where the variances $\{\sigma_n^2, n=1, ..N\}$ and the elements of C are unknown.

This form of the covariance matrix is suggested by the airborne multi-antenna radar problem. The assumption of a single interference covariance matrix common to all range cells does not seem appropriate, since clutter reflectivity often varies significantly from range cell to range cell. However, some sort of statistical regularity across range cells is needed; otherwise, there are too many parameters to be estimated.

The derivation of the clutter covariance matrix for this problem is presented in Appendix A in order to highlight the many assumptions and approximations which are involved. The result is that the clutter covariance matrix has the form given in (2). The covariance matrix of interest is actually that of the interference, which consists of both clutter and thermal noise. This matrix takes the form

$$\Lambda_n = \sigma_n^2 C + \sigma_o^2 I$$

where σ_o^2 is the thermal noise power per complex sample. The covariance model used in this paper is valid only if the clutter return is much larger than thermal noise.

Derivation of ML Covariance Estimate

The joint probability density of N independent vector observations of interference, each having M complex elements, is

$$p(\underline{z}_1, \ldots \underline{z}_N) = \prod_{n=1}^{N} \frac{1}{\pi^M \sigma_n^{2M} |C|} \exp\left\{ -\frac{1}{\sigma_n^2} \underline{z}_n^* C^{-1} \underline{z}_n \right\}$$

The ML estimates maximize this expression, or equivalently its logarithm,

$$\ell = \ell n\, p(\underline{z}_1, \ldots \underline{z}_N) = -MN \ell n \pi - M \sum \ell n \sigma_n^2 - N \ell n |C| - \sum \frac{1}{\sigma_n^2} \underline{z}_n^* C^{-1} \underline{z}_n \qquad (3)$$

with respect to $\{\sigma_n^2 \geq 0,\ n=1,\ldots N\}$ and the matrix C. This maximization will first be performed with respect to σ_n^2, with all other parameters held fixed. The log likelihood ratio (LLR) can be written as

$$\ell = -MN \ell n \pi - N \ell n |C| + \sum \ell n \left[\frac{1}{\sigma_n^{2M}} \exp\left\{ -\frac{1}{\sigma_n^2} \underline{z}_n^* C^{-1} \underline{z}_n \right\} \right]$$

Each term in the sum has the form

$$\ell n (x^M e^{-ax}) \qquad a, x > 0$$

which has a unique maximum at $x = \frac{M}{a}$. Thus

$$\hat{\sigma}_n^2 = \frac{1}{M} \underline{z}_n^* C^{-1} \underline{z}_n$$

is the ML estimate of σ_n^2, if C is known.

Consider next the maximization with respect to the M x M matrix C, with the σ_n^2 fixed. Following Goodman[2], the LLR can be written

$$\ell = \text{constant} - N \ln |C| - N\,\mathrm{Tr}\{C^{-1}S\}$$

where $S = \frac{1}{N}\sum \frac{\underline{z}_n \underline{z}_n^*}{\sigma_n^2}$. Goodman's proof can then be applied unchanged to establish that

$$\hat{C} = S$$

is the maximum likelihood estimate of C, provided the σ_n^2 are known.

In fact, both C and the σ_n^2 are unknown. Consequently, the ML estimates must be obtained as the simultaneous solution of the equations

$$\hat{C} = \frac{1}{N} \sum \frac{\underline{z}_n \underline{z}_n^*}{\hat{\sigma}_n^2} \tag{4}$$

$$\hat{\sigma}_n^2 = \frac{1}{M} \underline{z}_n^* \hat{C}^{-1} \underline{z}_n \tag{5}$$

In general, the solution must be obtained numerically, by iteration. Start the iteration by assuming

$$\hat{C}_o = \frac{b}{N} \sum \underline{z}_n \underline{z}_n^*$$

choosing the real constant b so that $(\hat{C}_o)_{11} = 1$. Invert $\hat{C}_o$ and compute the $\hat{\sigma}_n^2$ according to (5), use these to compute a better estimate $\hat{C}_1$, rescale it, and continue until convergence, as indicated by a sufficiently small change in the σ_n^2, is obtained. The rescaling is possible because the parameters specified in the model appear always as products; any one of them may be specified arbitrarily. This can be seen in the likelihood equations (4), (5) by the fact that if $C, \underline{\sigma}^2$ constitute a solution, then so also do $bC, b^{-1}\underline{\sigma}^2$. The solution can thus be rescaled at any point in the iteration.*

For brevity, this iterative algorithm will be called Weighted Covariance Estimation (WCE).

* Alternatively, the superfluous parameter could have been specified at the outset. However, it is felt that this approach results in a somewhat smoother exposition.

A disadvantage of the algorithm is the large amount of computation required. At each stage of the iteration, a square matrix whose dimension is that of the observation vector must be inverted.

When the dimension of the observation is 2, special simplifications occur, and the solution can be made more explicit.

Special Case : M = 2

After maximization with respect to C, the LLR takes the form

$$\ell = \text{constant} - N\ell n|S| - M\sum \ell n\ \sigma_n^2$$

In the special case M = 2, the determinant of S can be written

$$|S| = S_{11}S_{22} - |S_{12}|^2 = \frac{1}{N^2}\left(\sum \frac{|z_{n1}|^2}{\sigma_n^2} \sum \frac{|z_{n2}|^2}{\sigma_n^2} - \left|\sum \frac{z_{n1}z_{n2}^*}{\sigma_n^2}\right|^2\right)$$

Let

$$\xi_n = \frac{z_{n2}}{z_{n1}} \quad , \quad w_n = \frac{1}{N}\frac{|z_{n1}|^2}{\sigma_n^2} \geq 0$$

The determinant becomes

$$|S| = \Sigma w_n \ \Sigma w_n |\xi_n|^2 - |\Sigma w_n \xi_n^*|^2$$

$$= \frac{1}{2} \sum_m \sum_n w_m w_n |\xi_m - \xi_n|^2 = \frac{1}{2} \underline{w}^T R \underline{w}$$

where

$$R_{mn} = |\xi_m - \xi_n|^2 \tag{6}$$

In terms of the new unknown parameters $\{w_n\}$ the LLR is given by

$$\ell = \text{constant} - N \ln \underline{w}^T R \underline{w} + 2 \sum \ln w_n$$

Setting the derivatives with respect to the w_m equal to zero yields the set of equations

$$w_m (R\underline{w})_m = \frac{1}{N} \underline{w}^T R \underline{w} \qquad\qquad m = 1,\ldots N$$

The solution is clearly independent of the scaling of $\underline{w}$.* Thus, it suffices to seek the solution of

$$w_m (R\underline{w})_m = 1 \qquad\qquad m = 1,\ldots N \qquad (7)$$

and scale the w's afterward so that

$$S_{11} = \hat{C}_{11} = \sum w_n = 1$$

Explicit solutions of (7) have been obtained for $N \leq 4$.

$N = 2$: Any two positive numbers.

$N = 3$: $w_1 : w_2 : w_3 = R_{23} : R_{13} : R_{12}$

$N = 4$: $w_1 : w_2 : w_3 : w_4 = (R_{23} R_{24} R_{34})^{\frac{1}{2}} : (R_{13} R_{14} R_{34})^{\frac{1}{2}} : (R_{12} R_{14} R_{24})^{\frac{1}{2}} : (R_{12} R_{13} R_{23})^{\frac{1}{2}}$

For $N \geq 5$, analytic solution becomes prohibitively complicated, and an iterative solution is again required. Assume initially that all the w's are equal to N-1 and solve equation (7) iteratively.

$$w_m(n+1) = \frac{1}{(R\underline{w}(n))_m} \qquad\qquad m = 1,\ldots N \qquad (8)$$

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* In the case N=2, these equations degenerate into identities. In this case, the LLR is independent of the w's. Any two numbers summing to 1 will serve equally well.

Renormalize after each iteration so that $\Sigma w_m = 1$, and continue until convergence is obtained.

Once the w's have been determined, the ML estimate of the matrix C can be computed.

$$\hat{C} = S = \begin{bmatrix} 1 & \frac{1}{N} \Sigma w_n \xi_n^* \\ \\ \frac{1}{N} \Sigma w_n \xi_n & \frac{1}{N} \Sigma w_n |\xi_n|^2 \end{bmatrix} \qquad (9)$$

This algorithm is considerably simpler than the one for general M because the weights are determined iteratively from the observations, without matrix inversion, and the normalized covariance estimate $\hat{C}$ need be computed only once.

<u>Outlier Rejection Property</u>

The special form taken by the likelihood equations for M = 2 makes possible an interesting heuristic explanation of the "outlier rejection" property of the WCE algorithm. This behavior explains its relative immunity to corruption by desired signals hidden in the interference.

The mechanism for outlier rejection lies in the iteration (8) for the weights. Suppose that the ratios ξ_n are all nearly the same, except for ξ_k, and that the iteration begins with uniform weights $w_n = 1/N$. The elements of the k^{th} row and column of the matrix R defined by (6) will be much larger than the other elements. This will cause the k^{th} element of the vector $R\underline{w}$ to be much larger than its other elements. The k^{th} weight for the next iteration is the reciprocal of this element and so is much smaller than the other weights. The outlier has thus been deweighted.

The presence of a signal in a particular one of the observations causes the ratio for that observation to differ from those of other observations containing no signal. This observation then contributes little to the estimate of the covariance matrix. When this cell is processed for detection using the estimated covariance, improved detection probability results.

If all observations are assumed to be statistically identical, there is clearly no reason to treat one differently from any other. Removal of the assumption allows this possibility. The fact that it results in greater immunity to desired signal suppression is entirely fortuitous, however, since no mention of such signals was made in the model. It may be possible to introduce the occasional presence of signals into the model in a statistical fashion and obtain an algorithm with even better performance.

It is conjectured that the algorithm has similar immunity for any value of M. However, no plausibility argument has been found, and no simulation or experimental results are available.

Convergence

The iterative procedure has been programmed for the special case $M = 2$ and no problems with convergence have been encountered. However, convergence has not been proven theoretically.

If the statistical model proposed here is valid, WCE will improve upon SMI in the sense of producing a larger value of the generalized likelihood ratio. The initial covariance estimate used to start the iteration is the SMI estimate, and each subsequent iteration step increases (or at least does not decrease) the likelihood ratio. This is not inconsistent, since SMI is based on a simpler model. The real issue is whether or not the WCE model is a better representation of the physical situation for the particular problem of interest.

Accuracy of the Estimator

As the WCE estimator itself is defined only implicitly, no direct analytical assessment of its accuracy seems possible. One can, however, evaluate the Cramer-Rao bound on the joint covariance matrix of the estimation errors in all the unknown parameters. This provides a lower bound for the covariance matrix of any unbiased estimate. If any estimate achieves this lower bound, it is the maximum likelihood estimate (WCE). Also, the ML estimate is asymptotically efficient; that is, it approaches the bound asymptotically as the number of observations becomes large.

The calculation of the Fisher information matrix J is outlined in Appendix B for the case M = 2. The simplest result is obtained by representing the unknown covariance matrix C in the form*

$$C = \begin{pmatrix} 1 & gre^{j\phi} \\ gre^{-j\phi} & g^2 \end{pmatrix} \tag{10}$$

The errors of primary interest are those in r and g, which turn out to be uncorrelated. Their variances are bounded by

$$\sigma_r^2 \geq \frac{(1-r^2)^2}{2N} \tag{11}$$

$$\sigma_g^2 \geq \frac{g^2(1-r^2)}{2N} \tag{12}$$

Note that the estimates are (potentially) much more accurate when the magnitude of the correlation coefficient r is near unity.

It can be shown that the Cramer-Rao bounds on the errors in estimating ϕ, r, g in the constant covariance case (SMI) are exactly the same as those just presented for WCE. This means that the fluctuations in scale of the covariance matrix do not, in principle, affect the accuracy with which the elements of the constant part can be estimated.

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*The ML estimates of these parameters are

$$\hat{\phi} = \arg \hat{C}_{12}, \quad \hat{g} = \sqrt{\hat{C}_{22}}, \quad \hat{r} = \frac{|C_{12}|}{\sqrt{C_{22}}}$$

Airborne MTI

Up to this point, the development of the WCE algorithm has been kept as general as possible. It will subsequently be evaluated and compared with SMI for a problem of particular interest, radar detection of slowly moving ground targets from a moving aircraft, using both simulated and real radar data. This section discusses this problem in some detail.

The motion of the radar beam causes ground clutter returns to have a doppler frequency shift which depends on their position within the beam. If the radar antenna beam is fairly broad in azimuth, ground clutter returns from some part of the beam will have the same doppler frequency as the target returns of interest, so that conventional doppler processing of returns from a single antenna cannot separate targets from clutter.

One solution is to add a second antenna displaced from the first along the direction of motion of the platform, so that the two antennas occupy the same position in space (or nearly so) at different times. Pulses transmitted at these times yield nearly identical (i.e., highly correlated) returns from stationary objects, while returns from moving objects differ because of the motion. Subtraction of the returns received by the two antennas thus eliminates most of the ground clutter, leaving only returns from the moving objects.

For simplicity, only processing of pairs of returns, one from each antenna, in multiple range cells was simulated. The covariance matrix of a pair of returns from the n^{th} range cell is shown in Appendix A to be of the form

$$\Lambda_n = \sigma_n^2 C = \sigma_n^2 \begin{pmatrix} 1 & C_{12} \\ C_{12}^* & C_{22} \end{pmatrix}$$

The signal to be detected is of the form

$$\underline{s}^* = \beta \left\{ 1, e^{-j\phi} \right\}$$

where β is an arbitrary complex amplitude. An optimum processor for detecting this signal computes $|\underline{w}^*\underline{z}|$ and compares it to a threshold, where $\underline{w} = \Lambda_n^{-1}\underline{s} = C^{-1}\underline{s}$. Thus

$$\underline{w}(\phi) = \Lambda_n^{-1}\underline{s}(\phi) = C^{-1}\underline{s}(\phi) = \begin{bmatrix} C_{22} - C_{12}e^{j\phi} \\ -C_{12}^* + e^{j\phi} \end{bmatrix}$$

It is convenient and causes no degradation to scale the weighting vector so that $w_1 = 1$,

$$w_2(\phi) = \frac{-C_{12}^* + e^{j\phi}}{C_{22}-C_{12}e^{j\phi}} = -\frac{1}{C_{12}}\left(1 - \frac{C_{22}-|C_{12}|^2}{C_{22}-C_{12}e^{j\phi}}\right) \tag{13}$$

and detection is based on $|z_1 + w_2^* z_2|$. If the clutter is highly correlated ($C_{22} - |C_{12}|^2 << 1$) and if $C_{22}-C_{12}e^{j\phi}$ is not too small, i.e., if the signal is sufficiently different in doppler from the clutter, then

$$w_2 \cong -\frac{1}{C_{12}} \tag{14}$$

This means that the optimum weighting (13) which depends on the unknown signal phase ϕ, can be approximated by one which depends only on the statistics of the clutter, not on the nature of the desired signal.

The output signal-to-interference ratio (SIR) of a linear processor with weighting vector $\underline{w}$ is

$$\rho = \frac{|\underline{w}^*\underline{s}|^2}{E|\underline{w}^*\underline{z}|^2}$$

$$= \frac{|\underline{w}^*\underline{s}|^2}{\underline{w}^*\Lambda\underline{w}}$$

For the optimum choice of $\underline{w}$, the output SIR is

$$\rho^* = \underline{s}^* \Lambda^{-1} \underline{s} = \frac{|\beta|^2}{\sigma_n^2} \frac{1 + C_{22} - 2\mathrm{Re}(C_{12} e^{j\phi})}{C_{22} - |C_{12}|^2}$$

The quantity $|\beta|^2/\sigma_n^2$ is the input SIR (on antenna 1), so the improvement in SIR provided by the optimum processor is

$$I_{opt} = \frac{1 + C_{22} - 2\mathrm{Re}(C_{12} e^{j\phi})}{C_{22} - |C_{12}|^2} = \frac{|1 - C_{12} e^{j\phi}|^2}{C_{22} - |C_{12}|^2} + 1$$

This shows that the SIR improvement is quite large when the clutter is highly correlated ($C_{22} - |C_{12}|^2 << 1$). A similar calculation for the approximately optimum, Doppler-independent processor shows that its SIR improvement is

$$\tilde{I} = I_{opt} - 1$$

a negligible difference in performance when I_{opt} is large.

The cancellation ratio is defined to be the ratio of input to output interference powers of the canceller.

$$r = \frac{|w_1|^2 \; E\,|z_1|^2}{E\,|\underline{w}^* \underline{z}|^2}$$

The presence of $|w_1|^2$ in the numerator is equivalent to normalizing the weights so that $w_1 = 1$.

For the optimum processor

$$r_{opt} = \frac{C_{22}}{C_{22} - |C_{12}|^2} (1 - I_{opt}^{-1})$$

which is a (weak) function of the Doppler phase, while for the approximate processor

$$\tilde{r} = \frac{|c_{12}|^2}{c_{22} - |c_{12}|^2} \qquad (15)$$

independent of Doppler.

Simulation Results

The WCE algorithm was compared with SMI using simulated data. The basis of comparison was the cancellation ratio (15).

Simulated data were generated in 40 range cells. The power levels (i.e., the $\sigma_n 2$) in the cells were determined by selecting at random 40 numbers between 20 and 50 dB. The results are shown in Table I. For each range cell, thirty-two independent pairs of complex Gaussian random numbers having the covariance matrix

$$C = \begin{bmatrix} 1 & 1-10^{-5} \\ 1-10^{-5} & 1 \end{bmatrix}$$

were then generated and scaled to the appropriate power level.

The SMI algorithm estimated C_{12} using the first 10 range cells and used the result to compute $z_1 - \hat{C}_{12}^{-1} z_2$ in all 40 cells. The input and output interference powers, $|z_1|^2$ and $|z_1 - \hat{C}_{12}^{-1} z_2|^2$, in each of the 40 cells were averaged over the 32 independent trials, and the results used to compute an empirical cancellation ratio for each cell. These ratios were plotted as functions of the input interference power.

The entire process was repeated using the adaptive algorithm proposed in this paper on the same data set.

Figure 1 shows the cancellation ratios produced from the data by the approximately optimum, nonadaptive processor (14) which knows a priori the interference statistics. The empirical cancellation ratios are clustered about the theoretical value of 47 dB. This performance represents an upper bound for the adaptive techniques.

TABLE I

CLUTTER POWER (DB) IN EACH OF 40 RANGE CELLS

1	34.83	11	25.29	21	26.57	31	31.88
2	21.25	12	21.83	22	39.46	32	43.14
3	32.59	13	40.98	23	28.10	33	22.56
4	22.19	14	49.44	24	23.20	34	23.66
5	38.70	15	23.54	25	42.41	35	29.24
6	49.65	16	25.06	26	41.62	36	29.84
7	37.89	17	26.47	27	48.08	37	41.04
8	32.43	18	37.96	28	23.92	38	38.58
9	25.59	19	40.02	29	31.60	39	44.51
10	32.24	20	39.97	30	30.92	40	27.08

The same data processed by the SMI adaptive algorithm using 10 cells produced the results shown in Figure 2. The cancellation has been reduced by about 3 dB by the adaptivity. Note, however, that cancellation was considerably improved in one range cell. This is range cell 6, which was one of the cells used to estimate the covariance, and has the largest power level of these 10 cells. It tends to dominate the SMI estimate, which weights the 10 cells equally.

Figure 3 shows the corresponding result for the WCE algorithm, also adapting on the first 10 range cells. There is virtually no degradation due to adaptation. In view of the increased computation required, however, this modest increase in performance is not deemed significant. The real advantage is revealed when signals are present in the learning sample.

To this end, the data were modified by the addition of a simulated target having $\phi = \pi$ and adjustable amplitude to range cell 5.

Figures 4 a,b show the effect on the cancellation performance of the SMI and WCE algorithms, respectively, of a 20 dB signal present in cell 5. The interference power in cell 5 is 38.7 dB, so the SIR is -18.7 dB. SMI performance has degraded by 15 dB, while that of WCE is unaffected (the isolated point is that for cell 5).

Table II lists the weights assigned to the 10 learning cells on each of the 32 independent trials by WCE. Cell 5 is consistently deweighted by 30 or 40 dB relative to the other cells. It is therefore not surprising that the presence of the signal has little effect on the covariance estimate.

The computations were repeated for a variety of signal levels, and again using only the first 5 range cells for adaptation. The results are summarized in Figure 5. WCE performance is not affected by the presence of a signal, no matter how strong. SMI performance degrades at the rate of about 0.9 dB in cancellation for each additional 1 dB of signal.

The performance of the WCE algorithm begins to degrade when targets are present in half of the range cells used for covariance estimation. As the number of targets increases beyond this point, WCE and SMI are both degraded by about the same amount.

TABLE II

WCE WEIGHTS FOR CELLS 1-10 (32 TRIALS)

MOVING TARGET IN CELL 5, AMPLITUDE 20 DB

CELL 1	CELL 2	CELL 3	CELL 4	CELL 5	CELL 6	CELL 7	CELL 8	CELL 9	CELL 10
5.0446E-01	1.8718E+00	7.0834E-01	1.0116E+00	1.3716E-04	1.3556E+00	2.5891E-01	1.9548E+00	4.3162E-01	1.8427E+00
1.3302E+00	1.6137E+00	5.0997E-01	7.6528E-02	1.0787E-03	1.4278E+00	1.6307E+00	6.4688E-01	1.2415E+00	1.5156E+00
5.6330E-01	1.9081E+00	2.0384E-01	1.0843E+00	2.1944E-03	1.5202E+00	1.3614E+00	9.2736E-01	4.5895E-01	1.9764E+00
7.3622E-01	1.5797E+00	1.4084E-01	1.1333E+00	4.1034E-04	1.0742E+00	1.8164E+00	1.9983E+00	3.9882E-01	5.1338E-01
1.7300E+00	1.0041E+00	6.8117E-01	1.9763E+00	4.2685E-04	1.0887E+00	1.4658E+00	2.4850E-01	1.1840E-01	1.6857E+00
1.6532E+00	1.7148E+00	1.6357E+00	3.8097E-01	1.9086E-03	1.6359E+00	5.1778E-01	1.6156E+00	8.0432E-01	1.4026E-01
6.3647E-01	1.8012E+00	1.6309E+00	1.0153E-01	3.8347E-03	1.2969E-02	1.7575E+00	1.9246E+00	1.1439E+00	1.0869E+00
5.8244E-02	1.8207E+00	1.1039E+00	1.6901E+00	2.8647E-04	1.3506E+00	1.8262E+00	1.4396E+00	5.6889E-01	2.0676E-02
1.0768E+00	1.8478E+00	9.3018E-01	1.8917E+00	2.8334E-03	1.9392E-01	1.3762E+00	1.4572E+00	2.1964E-01	5.5077E-02
2.3484E-02	1.9857E+00	1.7790E+00	1.6195E+00	1.8598E-03	8.3445E-01	3.2151E-02	7.6842E-01	1.4155E+00	1.5599E+00
4.3386E-01	8.6452E-01	1.9648E+00	1.9151E+00	7.1736E-04	1.8686E+00	5.8122E-01	4.2938E-01	[illegible]	[illegible]
1.4486E+00	1.9769E+00	2.7038E-01	1.4759E+00	2.7102E-04	6.0082E-01	1.7445E+00	1.5935E+00	6.6113E-01	2.2731E-01
6.3605E-02	1.9558E+00	1.6637E+00	8.3310E-01	9.2844E-04	4.7675E-01	4.6234E-01	1.5883E+00	1.1294E+00	1.2258E+00
9.4237E-01	5.7593E-01	2.2283E-01	1.5425E+00	1.3967E-04	1.7563E+00	1.5761E+00	1.5288E+00	4.0715E-01	1.4412E+00
1.3075E+00	1.0154E+00	1.3422E+00	8.5151E-01	8.4022E-04	1.5002E+00	4.0765E-01	3.5813E-01	1.9506E+00	1.1760E+00
2.8163E-01	1.4629E+00	1.8882E+00	1.0209E+00	2.0606E-06	1.4769E+00	1.5900E+00	8.6215E-02	8.3015E-01	1.3031E+00
3.6385E-01	1.0796E+00	1.7311E+00	2.5531E-01	2.7782E-04	1.6283E+00	1.9863E+00	1.5556E+00	8.1519E-02	1.3152E+00
1.8193E+00	1.9852E+00	1.6370E+00	4.4457E-01	2.4578E-03	7.0686E-01	7.1489E-01	1.6445E+00	6.3002E-01	1.5168E-02
7.0425E-01	7.1298E-01	1.5296E+00	1.5832E+00	1.2048E-05	6.4605E-01	1.9798E+00	4.5363E-01	1.2970E+00	1.0275E+00
6.8201E-01	5.2281E-01	1.0690E+00	1.5971E+00	1.7256E-04	7.8846E-01	1.9224E+00	1.3391E+00	1.7057E+00	5.3628E-01
1.0823E+00	1.8697E+00	1.2933E+00	7.3868E-02	9.5362E-04	1.0724E-01	2.2961E-01	1.8223E+00	1.8491E+00	1.0717E+00
5.6470E-01	9.3904E-01	1.9997E+00	1.0291E+00	1.3323E-04	1.3706E-01	5.0517E-01	1.7112E+00	1.3612E+00	1.7527E+00
1.8698E+00	9.9930E-01	4.2862E-01	4.5399E-01	6.6820E-06	4.1900E-01	1.9982E+00	1.6742E+00	1.0731E+00	1.6837E+00
1.7855E+00	1.7782E+00	1.7590E+00	1.2632E+00	1.4903E-04	9.2921E-01	5.8277E-02	1.2084E+00	3.7096E-01	8.4708E-01
3.7045E-01	1.6913E+00	1.4856E+00	1.7381E+00	3.0848E-04	9.3109E-01	1.3504E+00	3.8580E-01	2.5602E-01	1.7908E+00
1.4478E+00	1.8962E+00	6.5282E-01	4.7367E-01	5.3980E-04	1.3684E+00	7.7058E-01	3.9865E-01	1.7175E+00	1.3747E+00
1.7596E+00	1.3075E+00	1.7242E+00	1.0715E-01	1.5268E-04	1.2540E+00	1.2972E+00	1.7711E+00	2.3065E-02	7.5601E-01
1.7478E+00	5.0154E-01	1.1928E+00	1.6933E+00	8.2237E-03	1.3327E+00	1.7177E+00	1.2121E+00	2.8219E-01	2.2886E-01
7.3873E-01	1.3188E+00	1.4447E+00	4.8156E-01	9.0609E-04	1.0800E+00	1.3076E+00	1.8658E+00	6.6643E-01	1.0394E+00
1.0000E+00	7.7081E-01	1.0708E-01	1.0332E+00	1.5680E-03	1.4727E+00	1.9756E+00	2.2033E-01	1.0326E+00	4.4617E-01
8.7088E-01	1.8834E+00	1.4133E+00	1.1906E+00	1.6404E-04	1.8267E-01	1.5927E+00	7.5260E-01	1.8806E+00	1.8543E-01
1.0058E-01	1.8803E+00	8.1351E-01	4.2758E-01	8.7282E-04	1.6728E+00	1.0035E+00	1.3030E+00	1.4188E+00	1.8037E+00

EOF:

Experimental Results

The simulation compared the performance of WCE and SMI estimators using data which obeyed the WCE statistical model. It was to be expected, therefore, that the WCE estimates would be better. In fact, the iteration which leads to the WCE estimate can always be started with the SMI estimate, and subsequent iterations can only increase the likelihood ratio.

The most important advantage of WCE, as revealed by simulation, is its tendency not to suppress desired signals present in the learning sample. It is important to see whether or not this advantage is maintained when using real data which does not precisely match either the WCE or SMI statistical models. For this reason, the two algorithms were compared using real data from an airborne L-band MTI radar having two antennas. Data processing consists of adaptive cancellation of each pair of pulses (one from each antenna) followed by Doppler filtering via a 32 point FFT, on each of 40 range cells.

In most cases, the two algorithms perform equally well. However, in the example to be presented here, a moving target in range cell 15 (S/C = -12 dB) causes a significant difference in performance. The cancellation obtained with WCE using range cells 11-20 for covariance estimation is shown in Fig. 6. It is limited by the thermal noise power level P_N. If clutter is cancelled well below the residual noise level, the expected cancellation ratio is

$$r = \frac{E|z_1|^2}{E|z_1 - wz_2|^2} = \frac{P_I}{2P_N}$$

since $|w| \approx 1$, so that

$$r(\mathrm{dB}) = P_I(\mathrm{dB}) - P_N(\mathrm{dB}) - 3$$

The cancellation ratios should thus form a straight line with a slope of 1 when plotted against the interference power P_I (dB). This behavior can be seen in the data. The Doppler-filtered canceller output is shown in Fig. 7.

The results produced by SMI using this data are shown in Figs. 8, 9. Cancellation is degraded by about 9 dB. The flatness of the curve indicates that thermal noise is no longer the limiting factor. The Doppler-filtered residues show that the detectability of small moving targets has been reduced considerably. The difference in performance would probably be much greater were it not for the limit imposed on WCE by thermal noise.

SUMMARY

An algorithm for generating a maximum likelihood estimate of the covariance matrix of a vector process, using multiple observations, has been developed. It is based on the assumption that the covariance matrices of the observations are the same except for a scale factor. The resulting estimate is a generalization of the sample covariance matrix in that it is a weighted average of the sample covariance matrices of the individual observations. The weights are computed via an iterative procedure.

Simulation results show that the adaptive "optimum" filter based on the weighted estimate offers some improvement in interference rejection compared to that based on the sample matrix estimate. The main advantage of the weighted covariance estimate, however, is that it has far greater immunity to errors caused by the presence of desired signals in the observations used for covariance estimation. It is therefore valuable in situations where target-free interference observations cannot be guaranteed.

The major disadvantage of the technique is the large amount of computation required. It is best suited to problems in which the dimension of the vector observations is small.

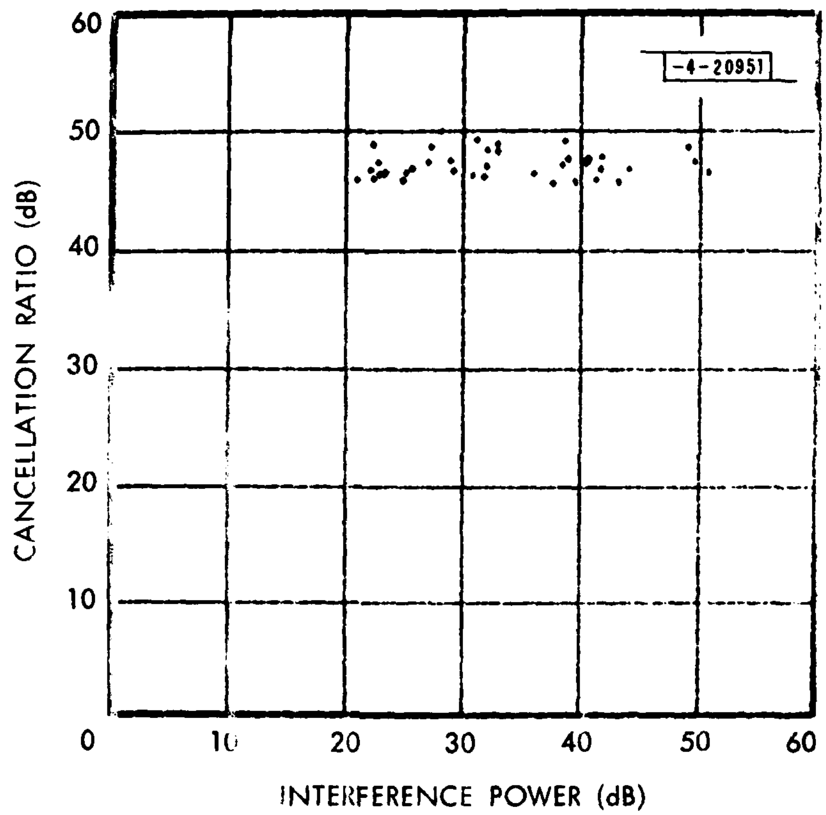

Fig. 1. Nonadaptive cancellation (known statistics).

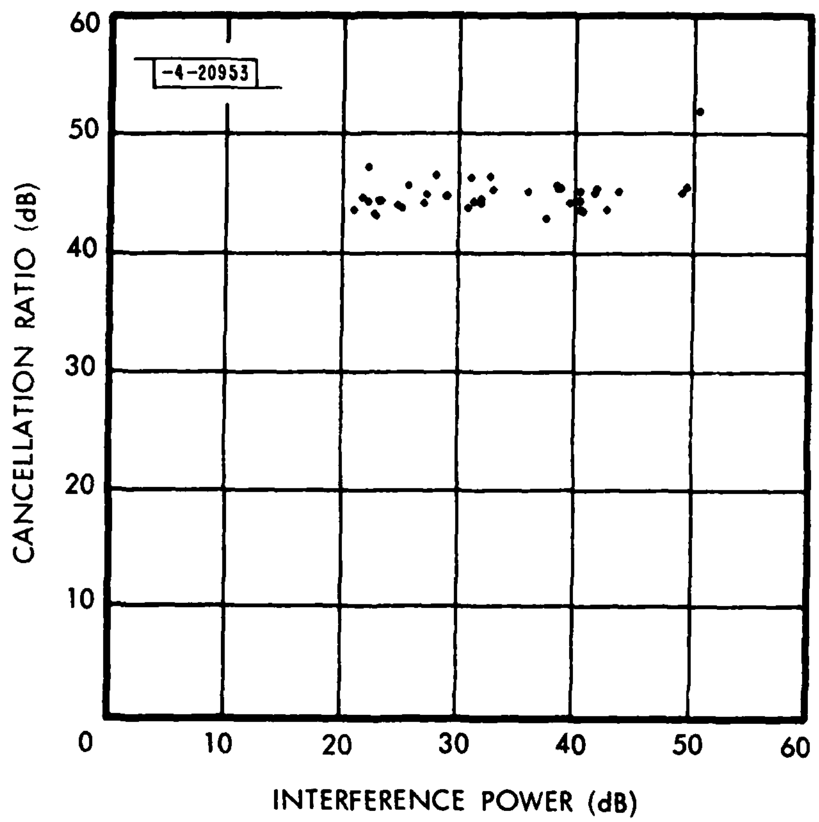

Fig. 2. Adaptive cancellation using SMI (10 cells).

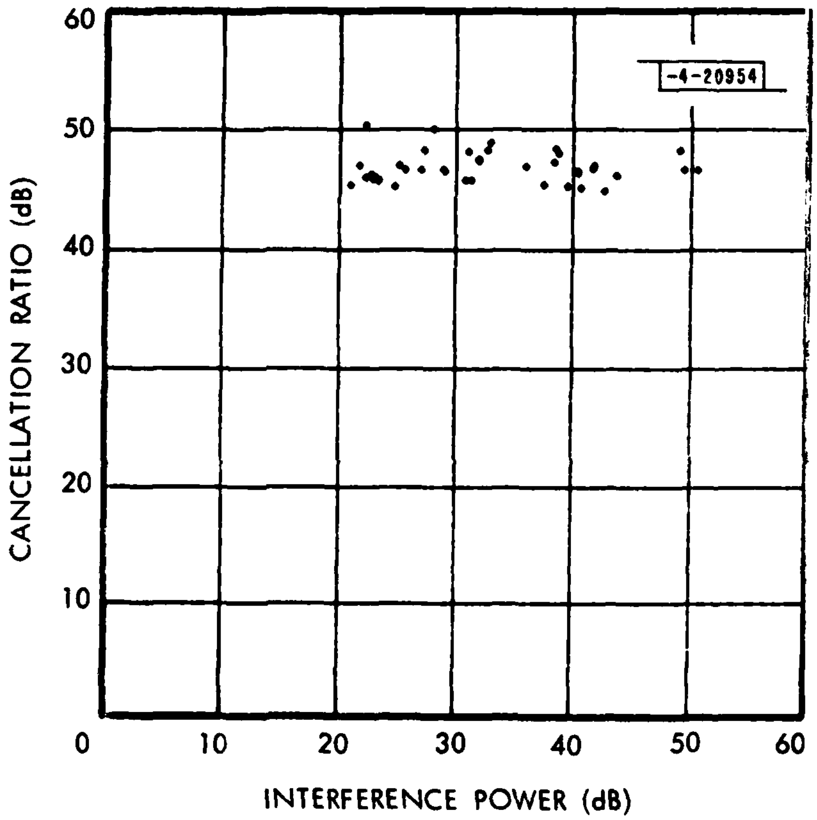

Fig. 3. Adaptive cancellation using WCE (10 cells).

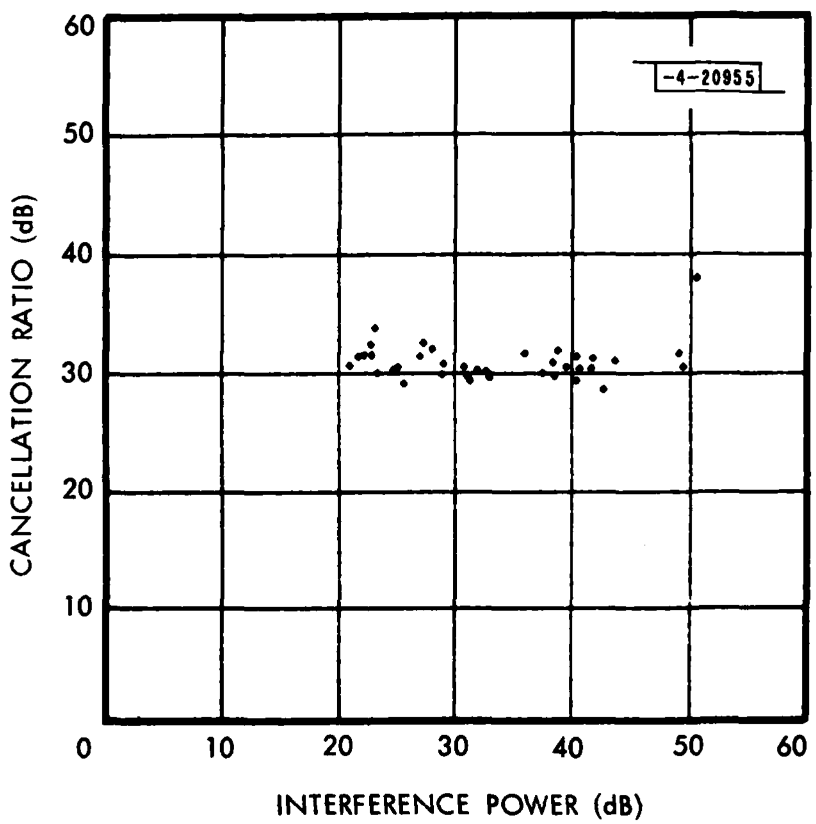

Fig. 4(a). SMI cancellation with 20 dB signal present.

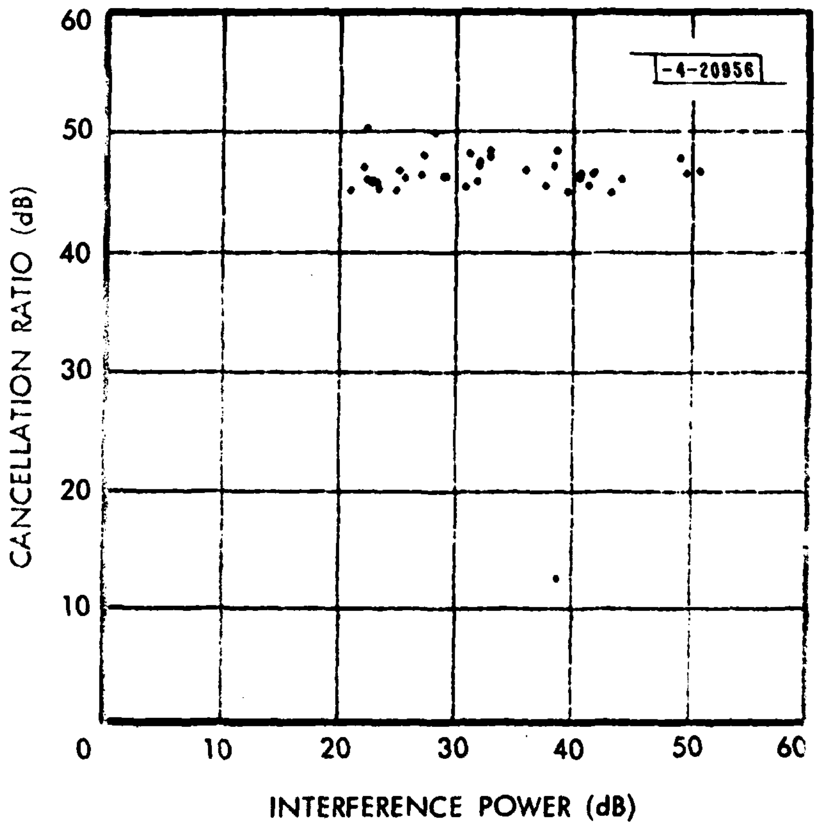

Fig. 4(b). WCE cancellation with 20 dB signal present.

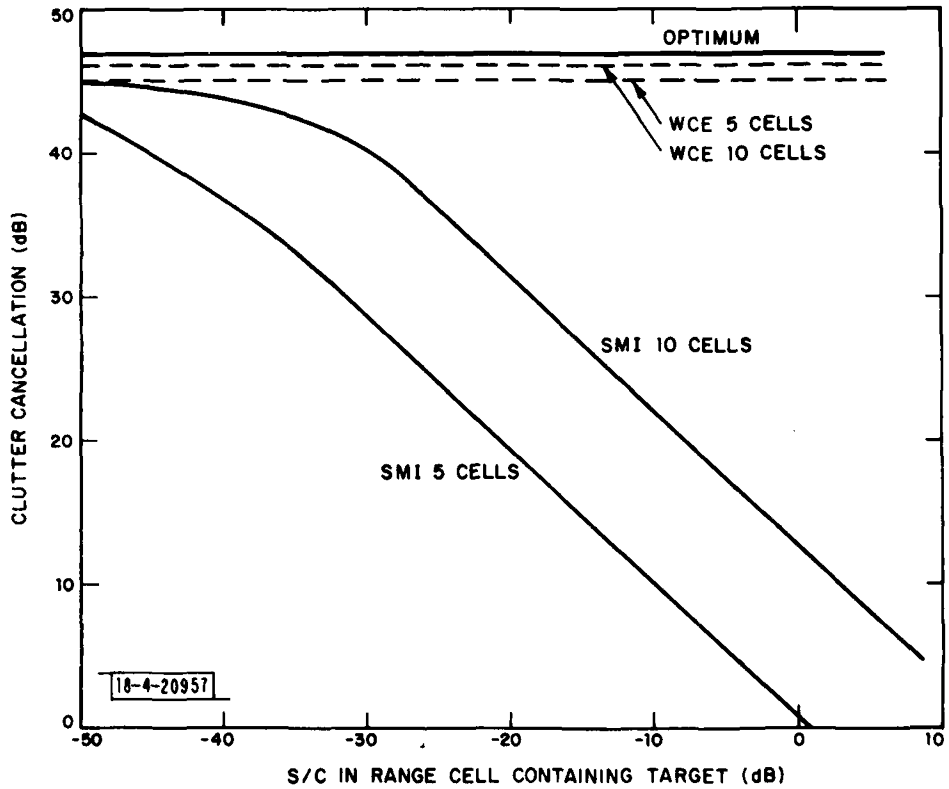

Fig. 5. Degradation in cancellation of SMI algorithm caused by presence of moving target in learning data.

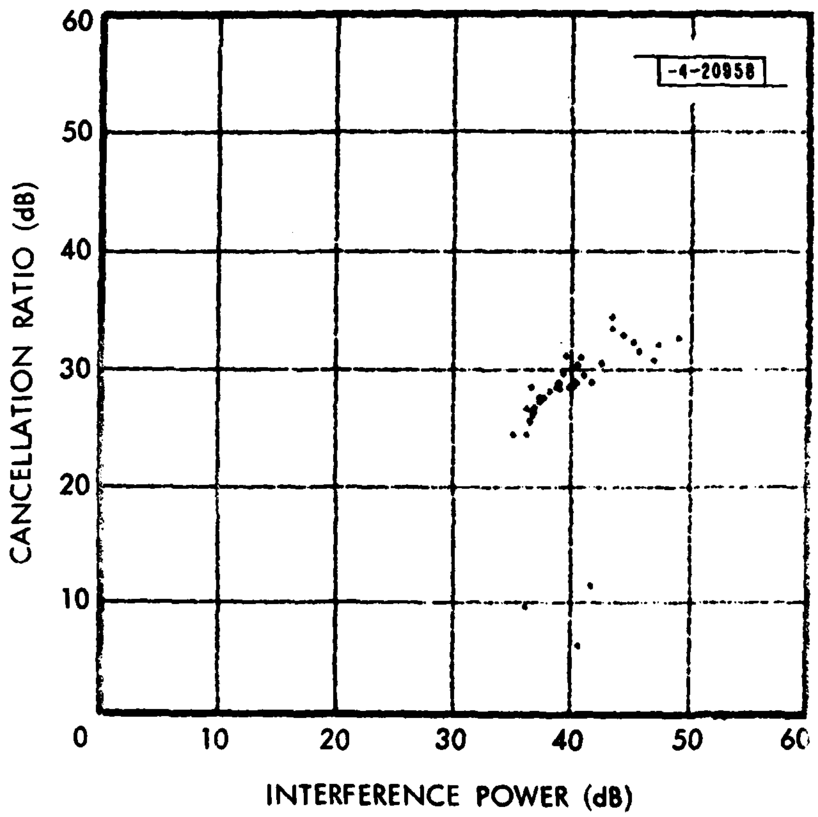

Fig. 6. Adaptive cancellation of real radar data using WCE algorithm.

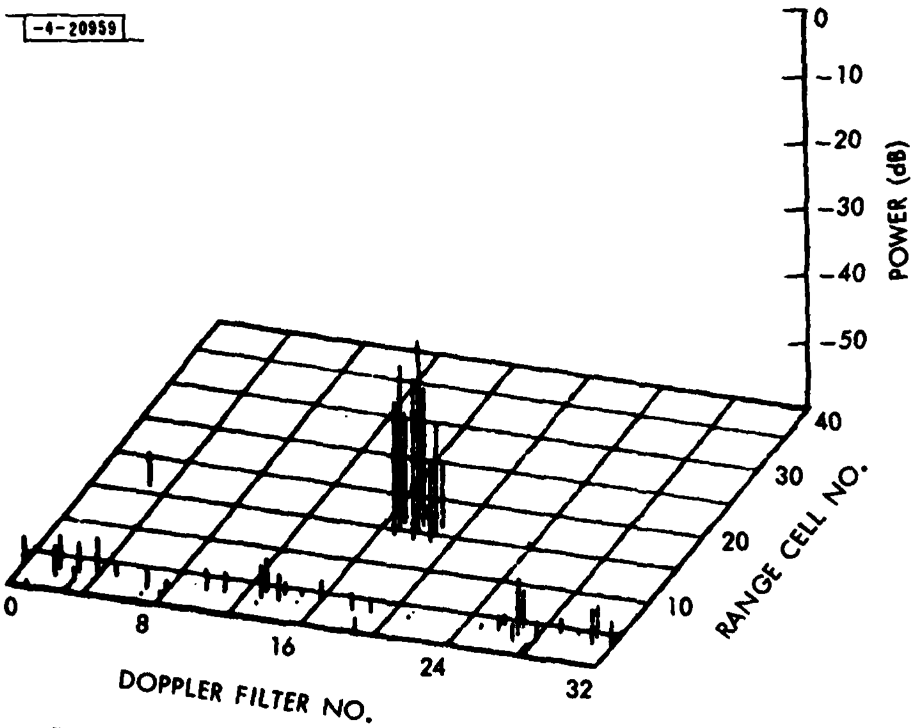

Fig. 7. Doppler filter outputs - WCE algorithm.

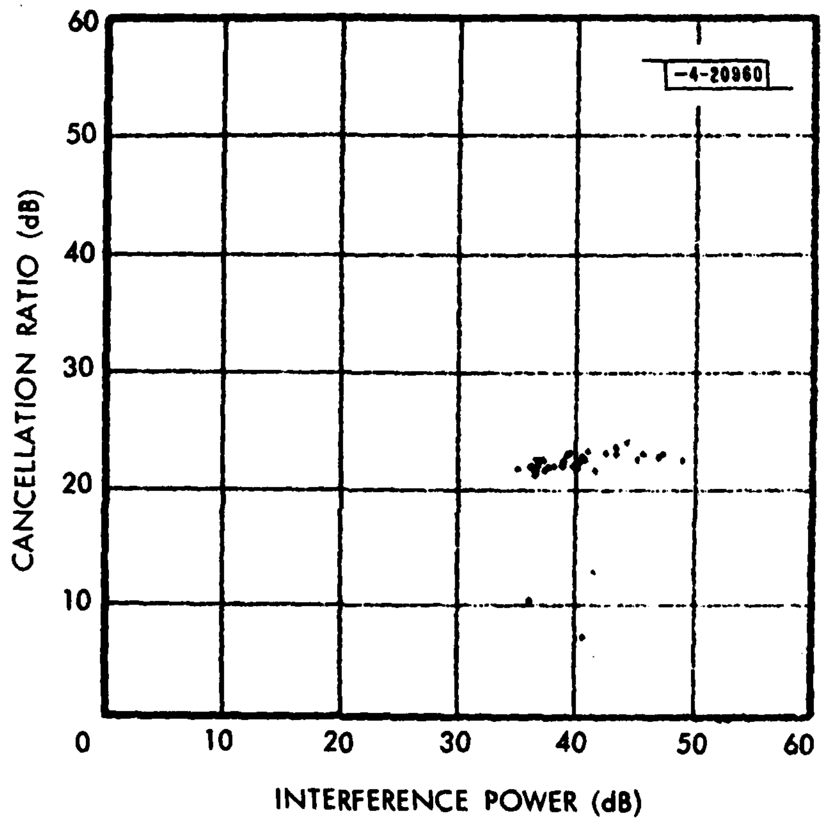

Fig. 8. Adaptive cancellation of real radar data using SMI algorithm.

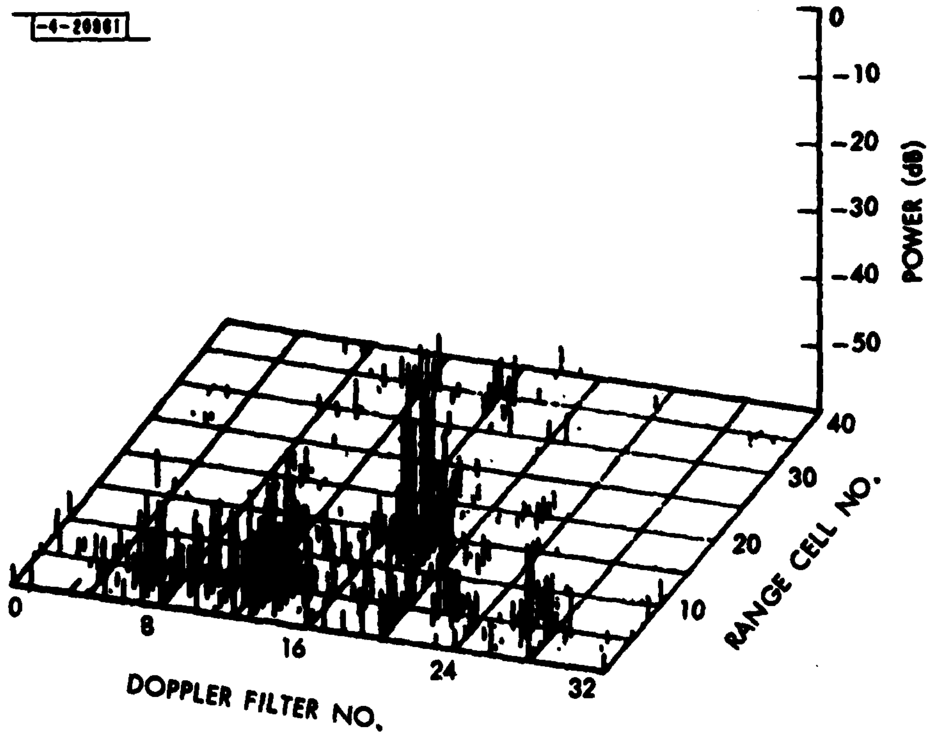

Fig. 9. Doppler filter outputs - SMI algorithm.

APPENDIX A

Derivation of Clutter Covariance Matrix for Multiple Antenna Airborne Radar

Consider an airborne radar having M antennas with phase centers located at positions $m\underline{d}$ with respect to some common reference point, moving with constant velocity vector $\underline{v}$. Let $\underline{R}_o$ be a vector from the reference point to an incremental clutter patch at coordinates r,θ at $t = 0$. The range vector from the m^{th} antenna to this patch at any time t is then (Fig. A1)

$$\underline{R}_m(t) = \underline{R}_o - m\underline{d} - t\underline{v}$$

For $m\underline{d}$ and $t\underline{v}$ sufficiently small, its length is approximately

$$R_m(t) \cong R_o - \frac{1}{R_o}(\underline{R}_o \cdot m\underline{d} + t\underline{R}_o \cdot \underline{v}) \triangleq R_o - md + \dot{R}_o t \qquad \text{(A1)}$$

Note that R_o, d and $\dot{R}_o$ are all *functions of r and θ*.

Let $\alpha(r,\theta)$ denote the complex amplitude of the return from the clutter patch at (r,θ), and $G_m(r,\theta)$, the complex gain of the m^{th} antenna in that direction. Assume that the m^{th} antenna transmits a pulse at time t_m.

$$s_m(t) = \mathrm{Re}\{u(t-t_m)\exp(j2\pi f_o t)\}$$

It is assumed that only the transmitting antenna receives the resulting returns, and that all returns are received before the next transmission from any other antenna occurs. With these assumptions, the total clutter return received by the m^{th} antenna is*

$$r_m(t) = \mathrm{Re}\left\{\iint \alpha G_m^2\, u(t-t_m-\tau(t))\exp\left\{j2\pi f_o(t-\tau(t))\right\} dr\,d\theta\right\} \qquad \text{(A2)}$$

where $\tau(t)$ is the delay suffered by an incremental element of return received at time t[4]. The behavior of $\tau(t)$ can be quite complicated, in general, but

*The (r,θ) dependence of α, G_m, and $\tau(t)$ has been suppressed for brevity.

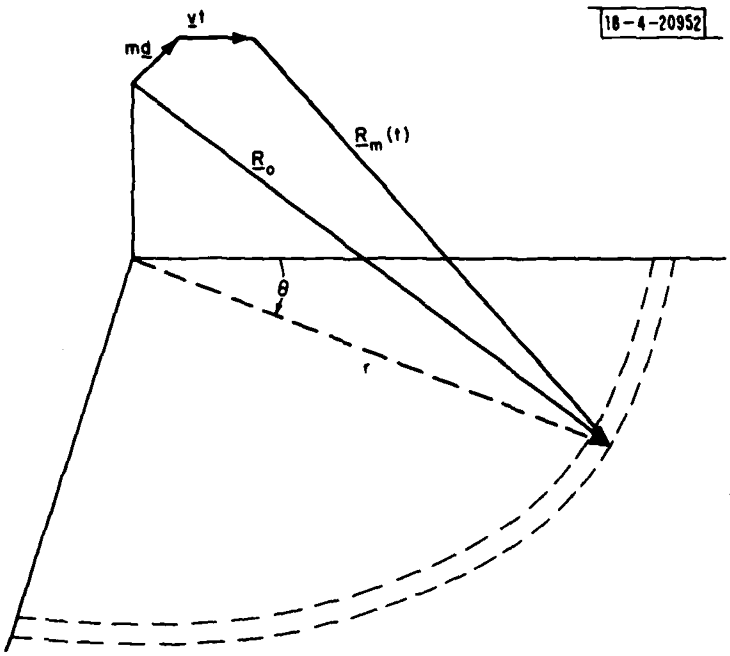

Fig. A1. Geometry of clutter return calculation.

using the linear approximation (A1) it can be shown that $\tau(t)$ is adequately approximated by

$$\underline{e}_R = \frac{\underline{R}_o}{R_o} \quad \tau(t) \cong \frac{2}{C}(R_o - md + \dot{R}_o t) = \tau_m + \dot{\tau}_o t$$

where $d = \underline{e}_R \cdot \underline{d}$, $\dot{R}_o = \underline{e}_R \cdot \underline{v}$. Using this and neglecting the effect of time compression on the complex modulation $u(t)$, equation (A2) reduces to

$$r_m(t) = \mathrm{Re}\left\{\iint \alpha G_m^2 \, u(t-t_m-\tau_m) \exp\left\{j2\pi f_o(t-\tau_m-\dot{\tau}_o t)\right\} dr d\theta\right\}$$

This return is passed through a filter matched to the transmitted pulse shape, yielding

$$C_m(t) = \int r_m(\tau) \, \mathrm{Re}\{u(\tau-t) \exp\{j2\pi f_o t\}\} d\tau$$

$$\cong \frac{1}{2} \mathrm{Re}\left[\iint \alpha G_m^2 \, \chi(t-t_m-\tau_m, f_d) \exp\{j2\pi f_d(t_m+\tau_m)\} \exp\{-j2\pi f_o \tau_m\} dr d\theta \exp\{j2\pi f_o t\}\right]$$

where the Doppler frequency $f_d = -f_o \dot{\tau}_o$ and τ_m are functions of (r,θ) and

$$\chi(t,f) \triangleq \int u(\tau) u^*(\tau-t) \exp\{j2\pi f \tau\} d\tau$$

is the ambiguity function of the radar pulse (see [4], p. 70). The complex modulation is sampled at time $t_m + \tau'$, yielding the complex clutter sample*

$$C_m = \iint \alpha G_m^2 \, \chi(\tau'-\tau_m, f_d) \exp\{j2\pi f_d(t_m + \tau_m)\} \exp\{-j2\pi f_o \tau_m\} dr d\theta$$

---

*The factor 1/2 has been absorbed into α.

It is now assumed that the maximum Doppler frequency of the clutter is much less than the bandwidth B of the radar pulse, making possible the approximation

$$\chi(t,f_d) \cong \chi(t,0)$$

The covariance of any pair of clutter samples is then given by

$$E(C_m C_n^*) = \iint P G_m^2 G_n^{*2} \, \chi(\tau'-\tau_m,0) \, \chi^*(\tau'-\tau_n,0) \tag{A3}$$

$$\exp\{j2\pi f_d(t_m-t_n)\} \quad \exp\{-j2\pi(f_o-f_d)(\tau_m-\tau_n)\} dr d\theta$$

It will be assumed that the transit time across the antenna configuration ($\max_{m,n} \tau_m-\tau_n$) is much less than the *reciprocal of the radar* bandwidth; the first argument of both ambiguity functions in (A3) can then be replaced by $\tau'-\tau_o$. Their product then becomes $|\chi(\tau'-\tau,0)|^2$. This is the envelope (squared) of the compressed radar pulse and is an impulse-like function of τ and therefore of r. The remainder of the integrand varies very slowly with r compared to $|\chi|^2$, so the r integration merely produces a constant ($\int|\chi|^2 dr$) which can be absorbed into P. Since it was already assumed that $f_d \ll B$, the last approximation also implies that $f_d(\tau_m-\tau_n) \ll 1$, so that this phase term may be neglected. Thus

$$E(C_m C_n^*) = \int P(\theta) G_m^2(\theta) G_n^{*2}(\theta) \exp\left\{ j2\pi [f_d(t_m-t_n) - f_o(\tau_m-\tau_n)] \right\} d\theta$$

where P, G, f_d, and $\tau_m-\tau_n$ are all implicitly functions of the sampling range

$R' = \frac{C\tau'}{2}$. Their variation with range is very *slow, however,* except for P. If the clutter samples are confined to a range interval which is small compared to the range, and if the clutter power distribution is the same in each range cell except for a scale factor, i.e., if

$$P(R',\theta) = F(R')Q(\theta)$$

then the covariance matrix of the clutter samples from a given range cell is simply a constant matrix multiplied by a scale factor which is a function of range.

Making the substitutions

$$t_m = m\Delta$$

$$f_d = -f_o\dot{\tau}_o = \frac{2}{\lambda}\,\underline{e}_R\cdot\underline{v}$$

$$f_o(\tau_m-\tau_n) = -\frac{2}{\lambda}\,\underline{e}_R\cdot\underline{d}(m-n)$$

the covariance element becomes

$$E(C_m\,C_n^{*}) = F(R')\int Q(\theta)G_m^2(\theta)G_n^{*2}(\theta)\,\exp\left\{j2\pi\frac{2}{\lambda}(m-n)(\underline{e}_R\cdot\underline{v}\Delta + \underline{e}_R\cdot\underline{d})\right\}d\theta \tag{A4}$$

The effective phase center location is defined to be the position of the m^{th} antenna at the time it transmits. If adjacent antennas transmit in sequence, the vector motion of the effective phase center between transmissions is then $\underline{d} + \underline{v}\Delta$. If the vectors $\underline{d}$ and $\underline{v}$ are collinear, this motion can be made zero by proper choice of Δ. If the antenna patterns are identical ($G_m \equiv G_n$), perfect clutter correlation is obtained. More generally, if the antenna patterns are real, as is usually the case, maximum correlation is obtained by choosing Δ to minimize the projection of $\underline{d} + \underline{v}\Delta$ on the line of sight $\underline{e}_R$ at beam center.

APPENDIX B

Outline of Derivation of Information Matrix

The elements of the Fisher information matrix are

$$J_{ij} = E\left\{\frac{\partial \ell}{\partial a_i} \frac{\partial \ell}{\partial a_j}\right\}$$

where ℓ is the likelihood ratio (3) and the a_i's are the unknown parameters. The constant part of the covariance matrix C is parametrized as in (10). Let x denote any one of these real parameters. Then

$$\frac{\partial \ell}{\partial x} = -\frac{N}{|C|}\frac{\partial |C|}{\partial x} + \sum \frac{1}{\sigma_n^2} \underline{z}_n^* C^{-1} \frac{\partial C}{\partial x} C^{-1} \underline{z}_n \tag{B1}$$

It will be convenient to define

$$\underline{\zeta}_n = \frac{1}{\sigma_n} C^{-1} \underline{z}_n$$

These are complex Gaussian random vectors with covariance C^{-1}. Using the relation[5]

$$\frac{\partial |C|}{\partial x} = |C| \operatorname{Tr}\left\{C^{-1}\frac{\partial C}{\partial x}\right\}$$

and replacing $\underline{z}_n$ by $\underline{\zeta}_n$, expression (B1) can be written

$$\frac{\partial \ell}{\partial x} = \sum_n \left(\underline{\zeta}_n^* \frac{\partial C}{\partial x} \underline{\zeta}_n - \operatorname{Tr}\left\{C^{-1} \frac{\partial C}{\partial x}\right\}\right)$$

Note that

$$E\left(\underline{\zeta}_n^* \frac{\partial C}{\partial x} \underline{\zeta}_n\right) = \operatorname{Tr}\left\{E(\underline{\zeta}_n \underline{\zeta}_n^*) \frac{\partial C}{\partial x}\right\} = \operatorname{Tr}\left(C^{-1} \frac{\partial C}{\partial x}\right) \tag{B2}$$

so that

$$E\left(\frac{\partial \ell}{\partial x}\right) = 0$$

Let y denote any one of the parameters of C. Then

$$E\left(\frac{\partial \ell}{\partial x}\frac{\partial \ell}{\partial y}\right) = E\sum_n \left(\underline{\zeta}_n^* \frac{\partial C}{\partial x}\underline{\zeta}_n - \mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}\right)\right) \cdot \sum_m \left(\underline{\zeta}_m^* \frac{\partial C}{\partial y}\underline{\zeta}_m - \mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial y}\right)\right)$$

$$= \sum_n \left[E\left(\underline{\zeta}_n^* \frac{\partial C}{\partial x}\underline{\zeta}_n\underline{\zeta}_m^* \frac{\partial C}{\partial y}\underline{\zeta}_m\right) - \mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}\right)\mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial y}\right)\right] \qquad \text{(B3)}$$

The following general result is useful in evaluating the above expression. Let A, B denote arbitrary square matrices and let $\underline{z}$ be a circular complex Gaussian vector with zero mean and covariance Λ. Then

$$E(\underline{z}^* A\underline{z}\ \underline{z}^* B\underline{z}) = E\left(\sum_{ij}\sum A_{ij} z_i^* z_j\right)\left(\sum_{k\ell}\sum B_{k\ell} z_k^* z_\ell\right)$$

$$= \sum_{ijk\ell}\sum\sum\sum A_{ij}B_{k\ell}E(z_i^* z_j z_k^* z_\ell)$$

The expectation can be evaluated *using Reed's results*[6]

$$= \sum_{ijk\ell}\sum\sum\sum A_{ij}B_{k\ell}(\Lambda_{ji}\Lambda_{\ell k} + \Lambda_{\ell i}\Lambda_{jk})$$

$$= \left(\sum_{ij}\sum A_{ij}\Lambda_{ji}\right)\left(\sum_{k\ell}\sum B_{k\ell}\Lambda_{\ell k}\right) + \sum_{ik}\sum\left(\sum_j A_{ij}\Lambda_{jk}\right)\left(\sum_\ell B_{k\ell}\Lambda_{\ell i}\right)$$

$$= \mathrm{Tr}(\Lambda A)\mathrm{Tr}(\Lambda B) + \mathrm{Tr}(\Lambda A\Lambda B) \qquad \text{(B4)}$$

Use of this result in (B3) results in

$$E\left(\frac{\partial \ell}{\partial x}\frac{\partial \ell}{\partial y}\right) = N\ \mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}C^{-1}\frac{\partial C}{\partial y}\right) \qquad \text{(B5)}$$

The other parameters σ_n^2 can be treated in a similar fashion. Using (3) for the case $M = 2$,

$$\frac{\partial \ell}{\partial \sigma_n^2} = -\frac{2}{\sigma_n^2} + \frac{\underline{z}_n^* C^{-1} \underline{z}_n}{(\sigma_n^2)^2} = \frac{1}{\sigma_n^2}(\underline{\zeta}_n^* C \underline{\zeta}_n - 2)$$

Again note that

$$E\left(\frac{\partial \ell}{\partial \sigma_n^2}\right) = \frac{1}{\sigma_n^2}(E(\underline{\zeta}_n^* C \underline{\zeta}_n) - 2)$$

$$= \frac{1}{\sigma_n^2}(\mathrm{Tr}(CC^{-1}) - 2) = 0$$

The elements of the information matrix corresponding to the σ_n^2 are

$$E\left(\frac{\partial \ell}{\partial \sigma_m^2} \frac{\partial \ell}{\partial \sigma_n^2}\right) = \frac{1}{\sigma_m^2 \sigma_n^2} E\{(\underline{\zeta}_m^* C \underline{\zeta}_m - 2)(\underline{\zeta}_n^* C \underline{\zeta}_n - 2)\}$$

$$= \frac{\delta_{mn}}{\sigma_n^4}[E(\underline{\zeta}_n^* C \underline{\zeta}_n \underline{\zeta}_n^* C \underline{\zeta}_n) - 4]$$

Use of (B4) results in

$$E\left(\frac{\partial \ell}{\partial \sigma_m^2} \frac{\partial \ell}{\partial \sigma_n^2}\right) = \frac{\delta_{mn}}{\sigma_n^4}[\mathrm{Tr}(C^{-1}CC^{-1}C) + \mathrm{Tr}(C^{-1}C)\mathrm{Tr}(C^{-1}C) - 4]$$

$$= \frac{2}{\sigma_n^4}\delta_{mn} \qquad \text{(B6)}$$

Finally, for the cross terms

$$E\left(\frac{\partial \ell}{\partial x}\frac{\partial \ell}{\partial \sigma_n^2}\right) = E\left\{\sum_m \left(\underline{\zeta}_m^* \frac{\partial C}{\partial x}\underline{\zeta}_m - \mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}\right)\right) \cdot \frac{1}{\sigma_n^2}\left(\underline{\zeta}_n^* C \underline{\zeta}_n - 2\right)\right\}$$

$$= \frac{1}{\sigma_n^2}\left[E\left(\underline{\zeta}_n^* \frac{\partial C}{\partial x}\underline{\zeta}_n \underline{\zeta}_n^* C \underline{\zeta}_n\right) - 2E\left(\underline{\zeta}_n^* \frac{\partial C}{\partial x}\underline{\zeta}_n\right)\right]$$

$$= \frac{1}{\sigma_n^2}\left[\mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}C^{-1}C\right) + \mathrm{Tr}(C^{-1}C)\,\mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}\right) - 2\,\mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}\right)\right]$$

$$= \frac{1}{\sigma_n^2}\mathrm{Tr}\left(C^{-1}\frac{\partial C}{\partial x}\right) \qquad \text{(B7)}$$

Evaluation of (B5), (B6), (B7) yields the information matrix J. If the vector of unknown parameters is ordered

$$\underline{a}^T = (\phi, r, g, \sigma_1^2, \ldots \sigma_N^2)$$

the result is

$$
J = 2\left[\begin{array}{ccc|ccc}
bNr^2 & 0 & 0 & 0 & \cdots & 0 \\
 & b^2N(1+r^2) & -bNrg^{-1} & -br\sigma_1^{-2} & \cdots & -br\sigma_N^{-2} \\
 & & bNg^{-2}(2-r^2) & g^{-1}\sigma_1^{-2} & \cdots & g^{-1}\sigma_N^{-2} \\
\hline
 & & & \sigma_1^{-4} & & \\
 & & & & \ddots & 0 \\
 & \text{symmetric} & & & & \sigma_N^{-4}
\end{array}\right]
$$

where $b = (1 - r^2)^{-1}$.

The phase estimation error is seen to be independent of all the other estimation errors. Its variance is bounded by

$$
\sigma_\phi^2 \geq \frac{1-r^2}{2Nr^2}
$$

The remaining portion of J can be written as

$$J' = 2D\left(\begin{array}{c|c} A & B \\ \hline B^T & I \end{array}\right) D \tag{B8}$$

where D is diagonal with $d_{11} = b$, $d_{22} = g^{-1}$, $\{d_{kk} = \sigma_{k-2}^{-2}$, $k = 3, .. N + 2\}$,

$$A_{2x2} = N \begin{pmatrix} 1+r^2 & -r \\ -r & \frac{2-r^2}{1-r^2} \end{pmatrix}$$

$$B_{2xN} = \begin{bmatrix} -r & -r & . & . & . & -r \\ 1 & 1 & . & . & . & 1 \end{bmatrix}$$

and I is the NxN identity matrix. Application of the formula for the inverse of a partitioned matrix to (B8) yields

$$(J')^{-1} = \frac{1}{2} D^{-1} \left[\begin{array}{c|c} (A - BB^T)^{-1} & -A^{-1}BQ^{-1} \\ \hline -Q^{-1}B^TA^{-1} & Q^{-1} \end{array}\right] D^{-1} \tag{B9}$$

where

$$Q = I - B^TA^{-1}B$$

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A new adaptive processing algorithm, called Weighted Covariance Estimation (WCE), for the detection of signals in interference of unknown character is presented. Its main advantage over present techniques (such as sample matrix inversion) is its tendency not to suppress desired signals present in the learning data.

WCE and SMI are compared for a radar problem of practical interest, adaptive MTI from a moving platform, using both simulated and actual radar data.